Nuclear charge distributions of ^{89}Y, ^{90}Zr and ^{92}Mo by elastic electron scattering

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Nuclear charge distributions of ^{89}Y, ^{90}Zr and ^{92}Mo by elastic electron scattering†

Abstract. Elastic scattering of electrons has been used to extract the nuclear charge distribution parameters of the $N = 50$ isotones; ^{89}Y, ^{90}Zr and ^{92}Mo. The momentum transfer range studied is from 0·25 fm^{-1} to 1·15 fm^{-1}. A comparison of the differences in the charge distributions is made with the shell model predictions.

Nuclear charge distribution parameters have been obtained using elastic electron scattering in various laboratories (Hofstadter and Collard 1967). As a gross property of the elements in the periodic table, the charge root mean square radius follows an $A^{1/3}$ dependence. The study of isotopes has revealed significant departures from this behaviour (Hofstadter *et al.* 1965, Singhal *et al.* 1970, Curran *et al.* 1971), and it is of interest to investigate the corresponding variations for a group of isotones (Sinha *et al.* 1971). If the additional protons in such a group are in the vicinity of a closed shell, the differences between the charge distributions should reflect the spatial distribution of these protons. This will be particularly true if the isotones have a closed major neutron shell. In this letter we present the results of our study of the isotone triplet, ^{89}Y, ^{90}Zr and ^{92}Mo with $N = 50$.

The isotone targets were in the form of metal foils of isotopic enrichment greater than 98% and a graphite target was used as a reference standard. The measurements were made with the Glasgow electron scattering facility described by Hogg *et al.* (1971) and the details of the data analysis can be found in Singhal *et al.* (1971).

An harmonic oscillator charge distribution, which includes corrections for the centre of mass motion and the finite proton size, was used for ^{12}C. The charge distribution parameters were taken from Bentz *et al.* (1967) and they are $a = 1·669$ fm and $\alpha = 1·006$. For the isotones, a two parameter Fermi distribution characterized by a half density radius c and a surface thickness t was used (Elton 1961). A Rawitscher–Fischer phase shift code was used to calculate all the cross sections, and

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a χ^2 function was minimized to obtain the optimum c and t values. To obtain the differences in the charge distributions, the ^{89}Y data were analysed with respect to ^{12}C, and the charge distribution parameters thus obtained were subsequently used in the analysis of ^{90}Zr and ^{92}Mo relative to ^{89}Y.

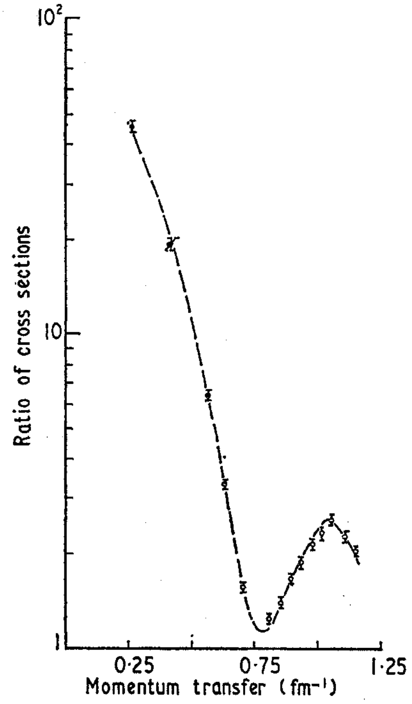

Figure 1. Comparison of the ^{89}Y and ^{12}C cross sections. The ^{12}C charge distribution parameters are taken from Bentz *et al.* (1967). ⫱ experimental points; – – – best fit curve.

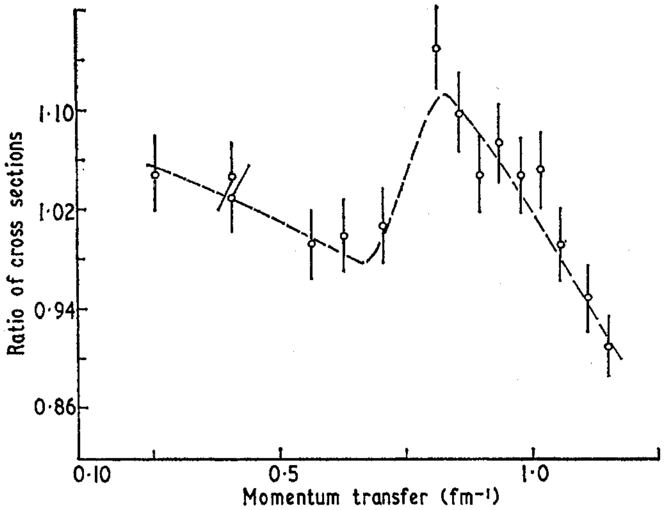

Figure 2. Comparison of the ^{90}Zr and ^{89}Y cross sections. ⫱ experimental points; – – – best fit curve.

Figure 1 gives the best fit for ^{89}Y with respect to ^{12}C, and figures 2 and 3 display the best fits for ^{90}Zr and ^{92}Mo. The best fit c and t values for the isotones are given in table 1. The errors in c and t were obtained by first constructing the χ^2 ellipses in the c, t plane and then applying the χ^2 criterion (Cline and Lesser 1970). To these uncertainties were added the target thickness normalization errors which were obtained by shifting the data points by 2% on either sides of their experimental values. The normalizations were performed at the lowest momentum transfer point of 0·25 fm^{-1}. In table 1, we have also listed values of the root mean square radii for ^{89}Y and ^{90}Zr which were obtained from the study of muonic atoms (Kessler *et al.* 1971, preprint). The agreement is very good.

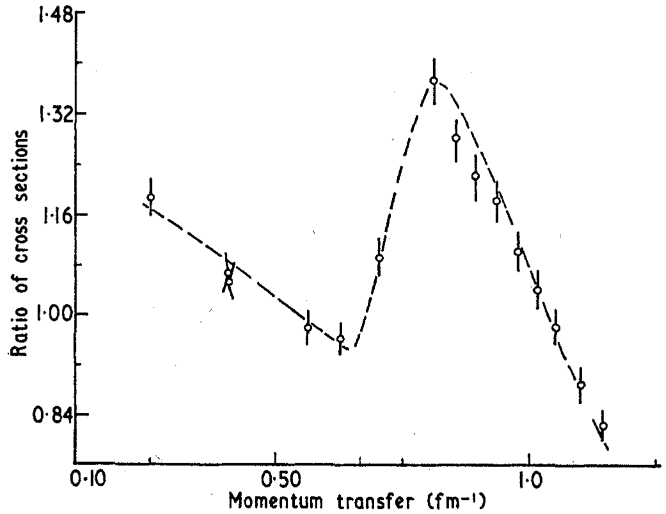

Figure 3. Comparison of the ^{92}Mo and ^{90}Zr cross sections. ϕ experimental points; – – – best fit curve.

Table 1. The charge distribution parameters of the isotones: ^{89}Y, ^{90}Zr and ^{92}Mo†

	c	t	Root mean square radius present	other
^{89}Y	4·814 ± 0·030	2·465 ± 0·015	4·272 ± 0·045	4·232‡
^{90}Zr	4·842 ± 0·025	2·543 ± 0·015	4·323 ± 0·040	4·267‡
^{92}Mo	4·912 ± 0·025	2·640 ± 0·015	4·411 ± 0·040	—

† All lengths are in fm. The errors quoted do not include the uncertainties in the parameters for the comparison nucleus. ^{12}C charge distribution parameters are $a = 1{\cdot}669$ fm and $\alpha = 1{\cdot}006$ (Bentz *et al.* 1967).

‡ Kessler *et al.* (1971).

It is interesting to compare the differences in the charge distributions between pairs of these isotones with the predictions of the shell model. The ground states of these isotones in a shell model scheme have been described in Alster *et al.* (1966) and in Bayman *et al.* (1959). Defining the harmonic oscillator parameter b from the

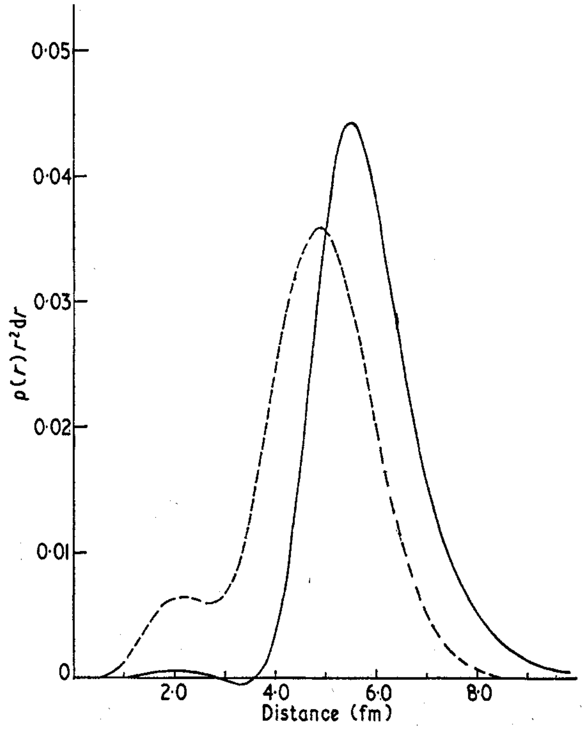

Figure 4. The difference in spatial distributions of charges in ^{90}Zr and ^{89}Y. The normalization is $\int\rho(r)r^2\,dr = 1$. Full curve, experimental; broken curve, shell model.

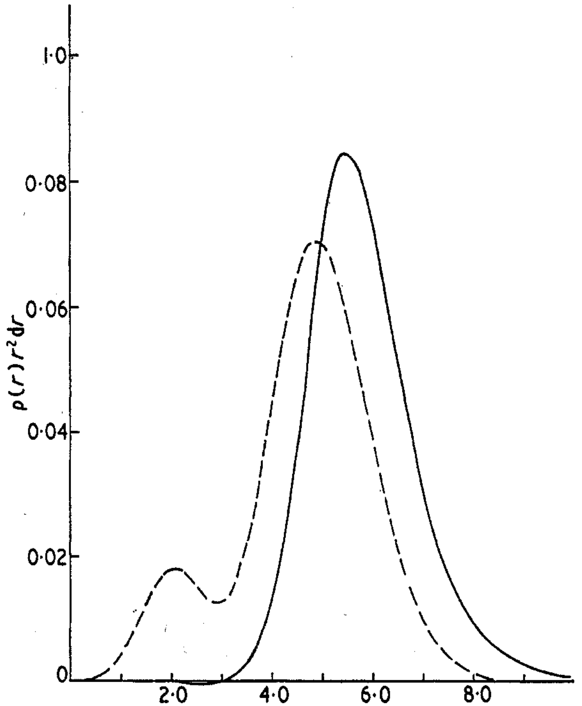

Figure 5. The difference in spatial distributions of charges in ^{92}Mo and ^{90}Zr. The normalization is $\int\rho(r)r^2\,dr = 2$. Full curve, experimental; broken curve, shell model.

equation, $\hbar\omega_0 = 41A^{-1/3}$, the distributions of the extra protons for the isotone pairs ^{90}Zr–^{89}Y and ^{92}Mo–^{90}Zr have been computed and are compared with the experimental predictions in figures 4 and 5 respectively.

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Light scattering by electrohydrodynamic fluctuations in nematic liquid crystals†

Abstract. We present some measurements on light scattered by a liquid crystal under a dc applied electric field. The results are interpreted in terms of radiation diffused by single scattering centres, which suffer electrohydrodynamic velocity fluctuations.

Hydrodynamic instabilities in nematic liquid crystals subjected to a dc electric field have been recently investigated (Heilmeier *et al.* 1968). This kind of effect can be directly observed by a microscope only when the applied electric field is low enough to ensure the presence of domain patterns (Durand *et al.* 1970). A slightly larger range of electric field intensities (some 10^3 V cm^{-1}) has been explored by measuring the 'rise time' of light scattering associated with induced instabilities

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